 

GCE/IAL applicable





Sequences, A.S. and G.S. (1)

GCE C1/IAL C12

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| Number and Algebra Strand (AS level) | | | |
| Learning Unit | Learning Objective | Time | Remarks |
| 1. Arithmetic and geometric sequences and their sums | 1. understand the concept and the properties of arithmetic sequences 2. understand the general term of an arithmetic sequence 3. understand the concept and the properties of geometric sequences 4. understand the general term of a geometric sequence 5. understand the general formulae of the sum of a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems 6. explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems 7. Introduction to recurrence relations 8. Communication between recurrence relations and AS,GS 9. solve related real-life problems | 17h | Some properties of arithmetic sequences :   * Ifis an arithmetic sequence, then is an arithmetic sequence   Some properties of geometric sequences :   * Ifis a geometric sequence, then is a geometric sequence   Example: geometry problems related to the sum of arithmetic or geometric sequences  Example: geometry problems related to infinite sum of geometric sequences.  Examples:  Interest, growth or depreciation. |

Main Concept 1:

General Sequences:

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| A sequence is a set of numbers (or patterns) arranged in a particular order.  Each number in the sequence is called a term of the sequence, the n-th term of the sequence is called the general term. |

Different kinds of Special Sequences:

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| 1. Arithmetic Sequences 2. Geometric Sequences 3. Square Sequences 4. Cubic Sequences 5. Triangular Sequences 6. Fibonacci Sequences 7. Other Special Sequences |

Remarks:

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| 1. General sequences appear in Paper II where you are provided with a sequence of numbers or patterns (usually dots or graphs). 2. Paper 2 skills:   To find the general term, you may substitute the first few terms and check if they can fit each of the numbers provided or derive it by the method of induction. (e.g. 1, 6, 11, 16, …）   1. Be careful: some general terms of special sequences involve (e.g. 5, 12, 21, 32, 45...) 2. Be careful: the sign of some general terms of the sequences. You may have to multiply (e.g. 3, 4, 5, 6, …) |

Some sequences are easy to identify

For example, 1,1,1,1,1,1,1,1,1……..

1,2,3,4,5,6,7,8…….

1,3,5,7,9,11,13,15,17…..

However, there is some sequence which are not easy to identify.

For example, 1,4,9,16,25,36,49…….

-1,4,-9,16,-25,36,-49…….

1,1,2,3,5,8,13,21,34,55…….

2,8,56…..

In this booklet, we will explore the properties of different kinds of sequences and develop problems.

Main Concept 2:

Arithmetic Sequences(A.S.):

**General term of A.S**

A.S. has some general properties:

usually, the general term is where *k* is an integer variable, and *a*, *c* are constants.

Let us take two sequences 1,1,1,1,1,1,1,1,1…… and 1,3,5,7,9,11,13,15,17……

For 1,1,1,1,1,1,1,1,1……, *a* is 1, and then *ck* = 0, so *c* = 0.

For 1,3,5,7,9,11,13,15,17……, *a* is 1 and then *ck* = 2, as the common difference is +2, so *c* is 2, the general term is 1 + 2*k*.

**Sum of an AS**

Carl Friedrich Gauss found the sum of the integers from 1 to 100 is 5 050. Gauss recognized he had fifty pairs of numbers when he added the first and last number in the series, the second and second-last number in the series, and so on. For example: (1 + 100), (2 + 99), (3 + 98), . . . , and each pair has a sum of 101.

Let us visualize his logic flow:

By reversing the sum,

By Observation, there is a total of 100 (*n*) terms together, and the individual sum is 101(*n* + 1). It would become

The sum(2*x*) is 101\*100=10100, then divide the sum equally to find *x*, it is 5 050.

Let us rewrite in general terms.

Note that there is nth terms in the above sequences, the individual sum is (n+1)ak and the total sum is 2y.

Therefore, .

Exercises:

Part I

Writing the next five terms:

1. 1, 2, 3, 4, 5, 6, 7……
2. 2, 4, 6, 8, 10, 12, ……
3. 4, 7, 10, 13, 16, 19, 22, ……
4. 3, 1, 1,3,5,7 ……
5. 11, 9, 7, 5, 3 ……

Part II

Find the common difference in the following sequences, write ‘no common difference’ if it is not an AS

1. 1, 1, 1, 1, 1, 1, 1, 1……
2. 2, 3, 4, 5, 6, 7, 8……
3. 1, 3, 5, 6, 7, 8, 9, 10……

1. 1, 4, 9, 16, 25……
2. 4, 8, 12, 16, 20, 24…….
3. 1, -1, -3, -5, -7, -9, -11……

Find the sum of the following A.S.

1. 1, 2, 3, 4, 5, 6, 7……200
2. 2, 4, 6, 8, 10, 12, ……600
3. 4, 7, 10, 13, 16, 19, 22, ……100
4. 3, 1, 1, 3, 5, 7 ……101
5. 11, 9, 7, 5, 3 ……
6. 1, 1, 1, 1, 1, 1, 1, 1……1
7. 2, 3, 4, 5, 6, 7, 8……18
8. 4, 8, 12, 16, 20, 24…….40
9. 1, 1, 3, 5, 7, 9, 11……17

Main Concept 3:

Properties of Arithmetic Sequences

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| 1. Proof: **Arithmetic Mean**   Given an arithmetic sequence , we have  ,  where *k* is a positive integer,  For *k* =1, we have   1. **Arithmetic Mean** 2. If (definition), then form an arithmetic sequence. 3. If for every positive integer *n* (definition), then form an arithmetic sequence.   **How to check whether a sequence is an A.S :**  **Check whether**   1. If form an arithmetic sequence,   then also form an arithmetic sequence, where *k* and *c* are constants.   1. The sum of two arithmetic sequences is an arithmetic sequence .   **Proof:** |